Advanced Research in Electrical and Electronic Engineering
Print ISSN: 2349-5804; Online ISSN: 2349-5812 Volume 1, Number 1 (2014) pp. 28-34

http://www.krishisanskriti.org/areee.html

Application of Wavelet – An Advanced Approach of Transformation

Mrinal Sharma[1], GagandeepSingh[2], Rajan Gupta[3]

[1, 2]*Department of Electronics & Communication, Lovely Professional University, Punjab*
[3]*Department of Computer Science, Faculty of Mathematical Sciences, University of Delhi, Delhi*

Abstract: **Wavelet transformation is an advanced method for resolving analysis problems in the domain of physics,medicine, engineering etc. The application of wavelet is not only restricted to wave signal processing in present scenario, but it is applicable in analyzing signalpropagation, pattern identification, computer vision, searching of planes and U-boats and various other biomedical technology fields.**

Wavelet Transform is actually a developedand more efficient version of Fourier transform. Fourier Transform is a powerful technique for analyzing stationary signals. In case of analysis of non-stationary or dynamic signal, their performance is severely affected due to the non-stationary interference as swept-tone interference persist.The Wavelet Transform on the other hand helps in taking care of components of a non-stationary signal. Wavelet Transform is able to work on complex data such as speech, images, tones and counterparts(patterns) to be resolved into various forms with high accuracy.

A wavelet packet decomposition technique is used to provide more accurate spectral estimation in signal processing and it effectively assertsnon-stationary interference sources.Signaltransmission is based on transmission of sequence of numbers (0 or 1). The sequence evaluationof a function is avitalthing, in all kind of signal transmission. The wavelet representation of a function is a new technique to be analyzed.

The major objective of this paper is to study the characteristics of wavelet transform in detail and compare it with most recent traditional form of transformation i.e. Fourier Transform. The paper also finds out the advantages of Wavelet Transform which are directly applicable for signal processing and communication systems compared to other transforms.The results of the study are simulated in MATLAB.Application of Wavelet transformation is found fruitful in Cognitive Radio which will be used for Spectrum Sensing in the upcoming years.

Keywords: Wavelet Transform, Fourier Transform,Cognitive Radio, Haar, DB3

1. INTRODUCTION

1.1 NEED OF TRANSFORMATIONS IN COMMUNICATION SYSTEM

Transformation means “re-expressions”. A researcher is simply re-expressing what the data have to say in other terms. However, it is necessary to diagnose the conclusions that we can draw on transformed data which may not always map to the original survey. Data transformations are commonly used tools which are usedin many functions for quantitative analysis of data in communication system .While these are important options for analysts, they do originally change the nature of the variable, making the explanation of the results few more complex. Data transformations are the petition of a mathematical modification to the values of a variable. There are a great diversity of available data transformations, from adding constants to multiplying or inverting and reflecting, and applying trigonometric transformations such as sine, or cosine wave transformations.A compiled benefit about most of the transformations is when we transform the data to meet one’s imagination. If one decide to transform, it is necessary to scrutinize that the variables are normally or nearly normally distributed after transformation. Then we can say that it worked. The basic idea of linear transformation in a communication system is as follows:

1. We can alter the coordinate system in which we indicates a signal in order to make it better suited for processing in the field of compression.

2. Important to dense the bulk of the energy (signal) into the some number of coefficients of the transforms.

3. We are capable to indicate all the necessary signal characteristics and important event in a dense manner.

1.2 CONVENTIONAL METHODS

There are various types of transforms that exists in mathematical space. They have different characteristics and properties associated with them. Some of the most common types of transforms are enlisted below. These transforms have been on an evolutionary path with Fast Fourier Transform being the most recent one in the list of traditional transforms.

1.2.1 DISCRETE COSINE TRANSFORM

The DCT [1] of data sequence X(m) ,m=0,1……..M-1 is defined as:

$G(K)=\sqrt{2}/M \sum_{m=0}^{m-1} X(m)*\cos(2m+1)K\pi/2M,$ (1)

Where, k=1, 2…………..M-1 and $G(0) =\sqrt{2}/M \sum_{m=0}^{m-1} X(m)$

Here G(0) is the 0th DCT coefficient and G(k) is the Kth DCT coefficient .

DCT coefficient means a set of basis vector that is:

$$\cos(2m+1)K\pi/2M$$

The discrete coefficient is a class of discrete Chebeshev polynomials, Where, Chebeshevpolynomials is:

$T(\in) =1/\sqrt{2}$ and $Tk(\in) =\cos(k\cos^{-1}(\in))$ (2)

Inverse DCT formula is given as:

$X(m) =1/\sqrt{2}\, G(0) + \sum_{k=1}^{m-1} G(k)\cos(\frac{(2m+1)K\pi}{2M})$` (3)

1.2.2 FOURIER TRANSFORM

If we have to operate on such sequences say X(i) for i=0,1,………N-1 it is obvious to develop a theory for them to determine particular orthogonal transformation which take sequence as X(i) into another sequence A(n) of same length as X(i) and which describes the frequency structure of X(i) this transformation is called as Discrete Fourier [2] and for X(j) it is defined as:

$A(n) = 1/N \sum_{j=0}^{N-1} X(j) * e^{-j2\pi\, n\, K/N}$ (4)

Similarly, if one have to find inverse formula:

$X(j) =\sum_{n=0}^{N-1} A(n)e^{j2\pi nK/N}$ (5)

1.2.3 FAST FOURIER TRANSFORM

A Fast Fourier transform (FFT) [3] is used to evaluate the Discrete Fourier Transform (DFT) and Inverse Fourier Transform. Fourier analysis transforms time domain to frequency and vice versa. A Fast Fourier Transform (FFT) evaluates such transformations by parceling out the DFT matrix into a product of sparse factors. Fast Fourier transforms have been stated as "the most necessary procedures of our life man ship". The DFT is obtained by dividing a sequence of values into components of various frequencies. A FFT is a route to evaluate the common result more quickly. FFTs are of great importance to a wide variety of applications, from DSP (digital signal processing)and resolving differential equations for quick multiplication of large integers. Several Fast Fourier Transforms algorithms only rely on the fact that $e^{-2\pi j/N}$ is an N^{th} fundamental root of unity, and thus can be used to transforms over any finite field, such as number-principled transforms. Since the inverse Discrete Fourier Transform is the same as the Discrete Fourier Transform, with the inverse sign in the exponent and a 1/N factor, any FFT algorithm can easily be conditioned for it.

Fourier transform and similar ones have a main disadvantage that information about only the frequency spectrum is provided and no information is available on the time in one dimension at which events happen. One solution to the issue of localizing transformation in the signal or image is to use the short time Fourier transform(STFT),where the signal is partitioned into small windows and treated locally as it were periodic.The uncertainty principle provides way on how to pick the windows to reduce minus effects.The window dilemma remains a narrow window yields poor frequency resolution, while a broad window gives a bad localization. The wavelet transform is usually compared with the Fourier transform. Fourier transform is a powerful method for analyzing the stationary signalbut is less useful in analyzing non-stationary signals. Non stationary are those where there is change in the properties of signal.Wavelet transforms permits the parts of a non-stationary signal to be analyzed and synthesized. That's why they are central theme of this study.

2. WAVELET TRANSFORM

The first use of wavelets was by Haar in 1909 [4]. He was keen in finding a basis on a dynamic space same as Fourier's basis in frequency space. In physics, wavelets were used in the characterization of Brownian motion. This work led to some of the opinion used to construct wavelet bases. If the features of the signal in question do not change over time, i.e, the signal is stationary then Fourier transform is substantial, for the analysis of the signal. Nevertheless, in many applications it is the variable or non-stationary phase of the signal (that is sudden changes) that is of maximum interest. In some cases, Fourier analysis is unable to find out when/where such events take place and is therefore not appropriate to depict or represent them. In order to conquer this limitation of Fourier to gain data in time and frequency domain,a different kind of transform, called wavelet transform can be used. Wavelet transform can be sighted as a trade-off between frequency and time domains. Fourier transforms a signal between time and frequency domains, while wavelet transform emphasizes on scales and locations (in place of frequency).

2.1 WAVELET

A wavelet means a small wave or we can say that a wavelet is an oscillation that decays speedily. Wavelet may be seen as an Integrant to modern Fourier decomposition method. Suppose, a particular class of functions is given and we want to search 'simple functions'

$f_0, f_1, f_2, \ldots f_n$ such that each

$f(x)= \sum_{n=0}^{\infty} a_n\, f_n(x)$ (6)

for some coefficients, a_n. Wavelet is a tool leading to representations of the type(5) for a big class of functions f. In1982 Jean Morlet, introduced the fact of the wavelet transform and provided a new tool for wave analysis. Morlet first considered wavelets as a sept or family of functions build from dilations methods or the translation methods of a particular function called the mother wavelet. Mother wavelet is defined by

$$\varphi_{a,b} = 1/\sqrt{|a|}(\ \varphi(t\text{-}b/a))\ a,b\in R\ ,\ a \neq 0 \qquad (7)$$

The numerical factor "a" is the scaling factor,and it computes the degree of compression. The factor b is the translation parameter which decides the time location of the wavelet. If |a| < 1, then the wavelet is in the compressed types of the mother wavelet and corresponds basically to higher frequencies. On the other hand, when |a| > 1, then $\varphi_{a,b}$has a larger time-width than $\varphi_{a,b}$ (t) and corresponds to lower frequencies. Thereby, wavelets have time-widths conditioned to their frequencies and this is the major reason for the usage of the Morlet wavelets in signal processing and signal analysis [5].

2.2 WAVELET TRANSFORM

There are different types of Wavelet transforms that are commonly used. They are as follows.

2.2.1 THE DISCRETE WAVELET TRANSFORM

The Wavelet Transform (WT) is a method for analyzing signals. It was developed as a replacement to the short time Fourier Transform (STFT) to conquer problems related to its frequency and time resolution characteristics. unlike the Short Time Fourier Transform that gives uniform time resolution for all Frequencies the Discrete Wavelet Transform gives high time resolution and low frequency resolution for high frequencies only and high level frequency resolution and low time resolution for low level frequencies. In that respect it is similar to the ear of a human which reveal similar time-frequency resolution properties.The Discrete Wavelet Transform (DWT) [5] [6] is a unique case of the Wavelet Transform (WT) form that gives a tight characterization of a signal in frequency and time that can be evaluated efficiently. The DWT is defined by the following equation-

$$w(j,k) = \sum_j \sum_k x(k) 2^{-\frac{j}{2}} \quad \varphi(2^{-j}(n-k)) \qquad (8)$$

whereφ (*t*) is a function of time with finite energy called the mother wavelet. The DWT analysis can be presented using a fast algorithm to multi rate filter banks (MFB).

2.2.2 CONTINUOUS WAVELET TRANSFORM –

In order to examine signals of very distinct sizes, it is necessary to use time-frequency atoms with different time pillars. The wavelet transform divided signals over extended and translated functions called wavelets, which transform a continuous function(CW) into a highly unnecessary function [5] [6].

A wavelet is a function with zero average formulated as follows-

$$\int_{-\infty}^{\infty} \varphi(t)\ dt = 0 \qquad (9)$$

Different types of wavelets have evolved with each one having different property and usage in different areas. Some examples of continuous wavelet areas follows.

2.2.2.1 MEXICAN HAT WAVELET

The Mexican hat wavelet [5] [6] is known as the second derivative of the Gaussian function g(t).

$$g(t)=1/\sqrt{2\pi}\sigma\ \ (\ e^{-t^2/2\sigma^2}) \qquad (10)$$

and second derivative is
$1/\sqrt{2\pi}\sigma^3\{\ (e^{-t^2/2\sigma^2})\ (t^2\ \ /\sigma^2\ \text{-}1)\}$

2.2.2.2 MORLET WAVELET

The commonly used Continuous Wavelet is the Morlet wavelet [5] [6], it is defined as following in time and frequency domains-

$$\varphi(t) = e^{imt} e^{-t^2\ /2} \pi^{-1/4} (11)$$

$$\varphi(\omega) = U(\omega)\ \ e^{-(\omega-m)^2\ /2} \pi^{-1/4} \qquad (12)$$

where, m is an adjustable variable of wave number and U is the step function that allows for correct signal reconstruction.

2.2.3 FAST WAVELET TRANSFORM

In 1988, Mallat developed a wavelet decomposition and reconstruction algorithm. This algorithm is for discrete wavelet transform (DWT) is really, a classical package in the signal processing community, known as a two-channel sub-band coder using quadrature mirror filters (QMFs).The DWT is defined for sequences with length of some power of two, and different ways of expanding samples of different sizes are needed. Methods for extending or expanding the signal include padding of zeros, periodic extension, and symmetry maintenance. The basic algorithm for the Discrete Wavelet Transform is not restricted to length and is based on simple operations of down-sampling or down-conversion and convolution. When a convolution is performed on finite length signals, distortions in boundary appear. To remove these effects in boundary, Fast Wavelet Transform(FWT) was introduced. This algorithm is a technique for the extension of a given signal with finite length.

2.3 COMMONLY USED WAVELET TRANSFORM

2.3.1 HAAR TRANSFORM

The Haar transform is the easiest of the wavelet transform. With the use of various stretches and shifts this transform slant multiplicationon a particular function against Haar Wavelet. It is same as the Fourier transform cross multiplied as a function against a sin wave with two phases and many stretches [7] [8].Haartransform is found to be more effective in applications such as signal and image compression in electrical and computer engineering as it gives a simple and computationally efficient advance approach for analyzing the local aspects of a signal.Haar transform is formulated as-

$$y_n = H_n x_n \qquad (13)$$

And the inverse formula is

$$x_n = H_n^T y_n \qquad (14)$$

Where H_n^T is the transpose Haar matrix.

A Haar transform decompose each signal into two components one is called average(approximation) & other is known as difference(detail). The formula for the average sub signal is as follows.

$$a_n = (.f_{2n-1}+f_{2n})/\sqrt{2} \qquad (15)$$

where n-=1,2,3………N/2

And the detail sub-signal part is given by:

$$a_n = (.f_{2n-1}-f_{2n})/\sqrt{2} \qquad (16)$$

where n-=1,2,3………N/2

Some properties of Haar transform are as follows-

No need for multiplications. It requires only additions and there are many elements with zero value in the Haar matrix, so the computation time is short. It is faster than Walsh transform, whose matrix is composed of +1 and -1.

Input and output lengths are the same. However,the length should be a power of 2.

It can be used to analyze the localized feature of signals. Due to the orthogonal property of Haar function, the frequency components of input signal can be analyzed.

2.3.1.1 FAST HAAR TRANSFORM

Fast Haar Transform (FHT) [7]involves subtraction, addition operation and division by two's operations, due to which it becomes faster and reduces the evaluation work in comparison to Haar Transform(HT).

2.3.1.2 MODIFIED FAST HAAR TRANSFORM

Modified Fast Haar Wavelet Transform (MFHWT), is one of the algorithms which can reduce the calculation work in Haar Transform (HT) and Fast Haar Transform (FHT). Haar transform is memory efficient, reversible without the edge effects, it is fast and simple. Fast Haar transforms includes subtraction, addition operations and division by two's operation also.Its application is found in image analysis, signal and image compressions. In this transform first average sub-signal at a one level for length N is:

$$a_m = (f_{4m-3}+f_{4m-2} + f_{4m-1} + f_{4m})/4 \qquad (17)$$

Where, n =1,2,3………N/4

$$d_m = ((.f_{4m-3}+f_{4m-2}) - (f_{4m-1} + f_{4m}))/4 \qquad (18)$$

Where, m= 1,2,3………N/4

Here four nodes are taking into the consideration instead of two nodes in HT and FHT [7] [8].

2.3.2 GAUSSIAN TRANSFORM

The direct Gaussian Transform [9] G in wavelet theory is defined as the operator which transforms or changes p(x) function into $G(\sigma^2)$, and the Inverse Gaussian Transform G^{-1} is opposite of G defined as the operator which maps $G(\sigma^2)$to p(x):

$$\int_0^\infty G(\sigma^2)\, N(x|\sigma^2) d\sigma^2 = p(x) \qquad (19)$$

Where,$N(x|\sigma^2)$is the gaussian distribution(Zero-mean) and it is formulated as

$$N(x|\sigma^2) = (1 / \sqrt{2\pi\sigma^2})\, e^{x^2/2\sigma^2} \qquad (20)$$

Where $G(\sigma^2)$ is the mixture of functions for to reproduce p(x).Properties of gauss transform is Final value theorem means transform is 0 when $\sigma^2$is tend to infinity.

2.3.4 DAUBECHIES WAVELET

Daubechies Wavelet is the group of the wavelets (orthogonal) denoting a Discrete Wavelet Transform(DWT). With every wavelet of this kind, there is a scale function or the father wavelet. Daubechies (DB) wavelets are basically used in solving a broad range of problems,as fractal problems, self-resemble properties of a signal etc.

3. DIFFERENCE BETWEEN WAVELET AND FOURIER TRANSFORM

Discrete Wavelet Transform is more informative and flexible than the other. It is a transform that breaks the data into frequency component or sub bands. Fourier involves the decomposition of a signal into sin waves of several frequencies. The advantage of the wavelet over Fourier is in analyzing physical situation that the sinusoid do not have a limited duration but instead extend from minus to plus infinity. In Fourier transform domain we completely lose information about the audio signal. A wavelet expansion coefficient refers a component that is local and easier to interpret. Wavelets are adjustable and adaptable and designed for adaptive systems whereas Fourier transform is suitable if the signal consists of few stationary components.

The Fourier transform shows up in a remarkable number of areas outside ofclassic signal processing. Even taking this into account, we think that it is safe to saythat the mathematics of wavelets is much larger than that of the Fourier transform. In fact,the mathematics of wavelets encompasses the Fourier transform. The size of wavelet theory is matched by the size of the application area. Initial waveletapplications involved signal processing and filtering. However, wavelets have beenapplied in many other areas including non-linear regression and compression. Some advantage of wavelet theory over Fourier is:

(a) A wavelet transform can be used to decompose or divide a signal into small wavelets and in wavelet theory, it is possible to obtain a good estimation of the given function f by using only a few coefficients which is a great attainment as compared to Fourier transform.

(b) One of the main advantages of wavelets is that they provide a concurrent fixing or localization in domain of time and frequency. Wavelets also use fast wavelet transform, so it is very fast.

(c) Wavelet transform can frequently squeeze or de-noise a signal in absence of considerable degradation.

(d) Wavelets have the advantage of being able to divide the pure details in a signal. Smaller wavelets can be applied to dissociate the most elementary details in a signal, while very large wavelets can identify other details of coarse analysis.

(e) Wavelet theory is competent to declare aspects of data that other signal analysis method misses. The features like breakdown points and segregation in higher order derivatives are perfect example for this.

The transforms were simulated in the MATLAB to study the behavior of each one of them exhibiting different properties. For Fourier series, in MATLAB Simulation first we construct a function x(n) which takes the input value after applying transformation's standard formula. In Figure 1, we have X(k) as Fourier transformed data which can be seen the plot.For the given data sequence [1 2 3], the Fourier transform is complex 6.0000, -1.5000 + 0.8660i and -1.5000 - 0.8660i by taking magnitude we can get the value as $\sqrt{x^2+y^2}$ where x and y are the real and complex part respectively.here X(k) is the fourier transform of the x(n).

For wavelet decomposition, we first apply signals to the low pass and high pass signals and then we perform the down-sampling by a particular factor. After this we have two parts and by taking only the low pass signal down-sample part and perform further decomposition on it, we again get two parts. We continued our analysis on the low pass down-sampled part as shown in Figure 2.

Here we caneasily differentiate between the Fourier and the Wavelet Transform.The later one is more informative because wavelet uses the decomposition stages and gives the information about the analysis but in Fourier we have only the output at a particular level. We can see that the output from LPF and HPF after down-sampling is a1 and d1 then we take only the LPF down-sampling part and again performing LPF and HPF operations now we have a2 and d2. After three level of decomposition we observe the wavelet a3 and d3.

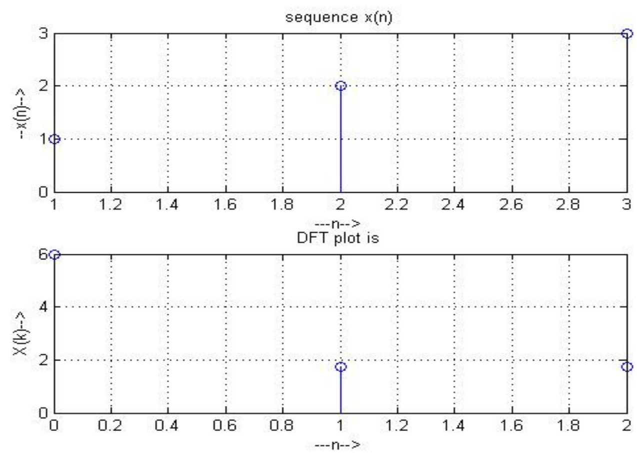

Fig. 1Simulated results for Discrete Fourier Transform

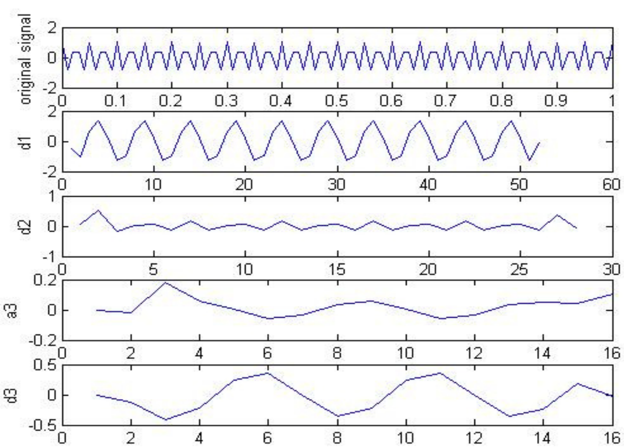

Fig. 2 Simulated results for Wavelet Transform

4. APPLICATION OF WAVELET TRANSFORM

The present research deals with the exploration & application of Wavelets to help in the simulation and analysis of non-stationary signal data. The coefficients of wavelets can be used in a variety of method to draw out helpful signal information. Wavelet coefficients may be used to receive an estimation of the power spectrum. The Wavelet coefficients also give the Scalo-gram, which tells the signal energy on a scaling time domain. The main advantages of wavelet methods over traditional Fourier methods are the use of localized basis functions and the faster computation speed. Localized basis functions are ideal for analyzing real physical situations in which a signal contains discontinuities and sharp spikes. This makes possible for identification of time-varying energy flux, and transient bursts that are not readily directed using the time or frequency domain methods. The property of accurate energy representation confers itself well to reconstruction of the signal with simulation. The trimming of noise in a measured signal may be fulfilled by changing wavelet coefficients below a specific threshold.

A variety of examples are provided herein to demonstrate these applications of wavelet transforms.

4.1 PHYSICS APPLICATION (SCALOGRAM)

In physics the scalo-gram gives much information about the nature of non-stationary processes. An example application is the interpretation of special events in behavior of the structural during earthquake. Any transformation in frequency content, e.g., beginning of stiffness deterioration, abrupt occurrence of unyielding events, energy swapping between modes by reaction coupling, or encompass between structural components can be identified by the scalo-gram. The wavelet scalo-gram of ground acceleration indicates the presence of high frequency energy early in the record which attenuates with time. The presence of high frequency energy will tend to excite higher structural modes. This information may be useful in predicting the building response modes that may be excited during similar earthquakes and the time such excitation begins. The monitoring of such performance information is not available via Short Time Fourier Transform due to the inflexibility of the time-frequency window which precludes identification of discontinuities in a signal. A multi-filter approach utilizing simple oscillators has been addressed by other researchers to avoid the STFT shortcomings and present time dependent frequency fluctuations.

4.2 ENGINEERING APPLICATION

(a) Wavelet filter-bank signal decomposition

The DWT is a convenient and efficient method of observing the execution of time dependent dynamic systems. While Fourier coefficients do not contain time related information, the coefficients depicting the localized basis functions do reflect time dependence. Wavelet domain representation is managed well as wavelet coefficients provide unique insights into transient events within a time series.

(b) Compression of Audio Data

In this approach the data is divided into small frames, for each frame a wavelet representation is used to minimize the number of bits requisite to represent the frame. The Audio codec utilizes the wavelet transformation for compression of a high quality audio whilst.

4.3 COGNITIVE RADIO APPLICATION

The original WDCS implementation (Wavelet domain communication system) uses a linear Wavelet-based transform to perform spectral estimation and was evolved to overcome two main TDCS shortcomings [10] which are:

a) The Fourier-based estimator instinctively dilates interference energy into proximate spectral domains not containing interference energy, an inefficiency probably resulting in less performance.
b) The TDCS is unable to efficiently estimate the spectral type of non-stationary interference.

The WDCS architecture exclusively fungibled the Fourier based spectral estimation processes with a lineal wavelet transform. And, the inverse Wavelet transform block replaced the inverse Fourier transform block. After scaling and translation we can achieve a two-dimensional mother wavelet

Simulations of different wavelet are conducted in MATLAB. Here we have three signals sine, pulse and delta signals. We want to perform the DWT operations on each signal,after adding the AWGN(additive white Gaussian noise) with them. The distorted sine,pulse and delta signals are shown in Figure 3, Figure 4 and Figure 5.

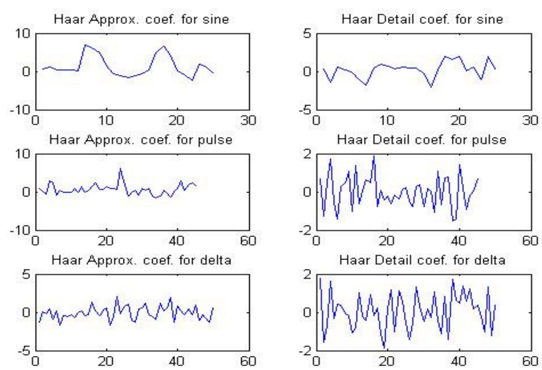

Fig. 3 Simulation using HAAR

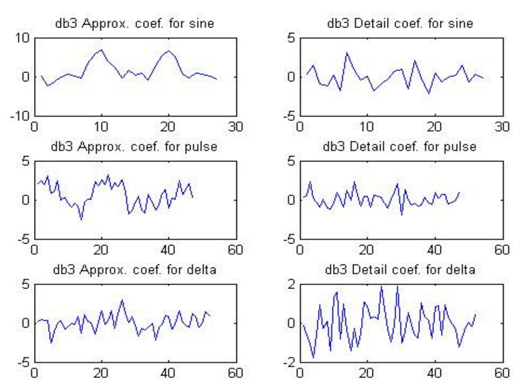

Fig. 4 Simulations using DB3

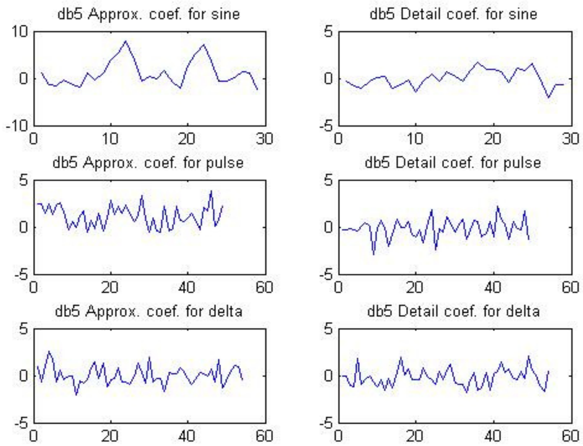

Fig. 5 Simulations using DB5

5. CONCLUSION

In this paper, we have discussed about applications & usage of Wavelets such as Data compression, Scalogram, Wavelet Filter-bank and Cognitive Radio usage with the help of Wavelet Transform approach. We have also presented a comparative discussion of Fourier Transform and Wavelet Transform mentioning the disadvantages of Fourier Transform which were followed by the advantages of wavelet transform. From the study it was evident that the Wavelet Transform based approach is much better than the existing methods and it is more informative. Finally, we can say that wavelet transform is a reliable and better technique than that of Fourier transform and other techniques. For upcoming concepts like cognitive radio, the main issue is to sense and identify all spectrum holes present in the environment. From this paper we can suggest a wavelet based approach WDCS rather than TDCS for spectrum sensing and spectrum Hole detection. The proposed sensing techniques provide an effective radio sensing architecture for WDCS to identify and locate spectrum holes in the signal spectrum.

REFERENCES

[1] Chitprasert, B., and K. R. Rao. "Discrete cosine transform filtering." Signal processing 19, no. 3 (1990): 233-245.

[2] Cooley, J., P. Lewis, and P. Welch. "The finite Fourier transform." IEEE Transactions on audio and electroacoustics 17, no. 2 (1969): 77-85.

[3] Heckbert, Paul. "Fourier Transforms and the Fast Fourier Transform (FFT) Algorithm." Computer Graphics 2 (1995): 15-463.

[4] Chun-Lin, Liu. "A tutorial of the wavelet transform." ed: NTUEE, Taiwan (2010).

[5] Polikar, Robi. "The wavelet tutorial." (1996).

[6] Heil, Christopher E., and David F. Walnut. "Continuous and discrete wavelet transforms." SIAM review 31, no. 4 (1989): 628-666.

[7] Bhardwaj ,Anuj, and Rashid Ali. "Image compression using modified fast haar wavelet transform." World Applied Sciences Journal 7, no. 5 (2009): 647-653.

[8] Porwik, Piotr, and AgnieszkaLisowska. "The Haar-wavelet transform in digital image processing: its status and achievements." Machine graphics and vision13, no. 1/2 (2004): 79-98.

[9] Alecu, TeodorIulian, SviatoslavVoloshynovskiy, and Thierry Pun. "The gaussian transform."In EUSIPCO2005, 13th European Signal Processing Conference. 2005.

[10] Lee, Marion J., Michael A. Temple, Roger L. Claypoole, and Richard A. Raines. "Transform domain communications and interference avoidance using wavelet packet decomposition." In Wireless Communications and Networking Conference, 2002. WCNC2002. 2002 IEEE, vol. 1, pp. 255-259. IEEE, 2002.